2D FRAME AND TRUSS ANALYSIS

Howe Truss Analysis: Worked Example

Four-panel Howe truss

Hand calculation, independent checks and comparison with the Optimal Beam 2D Structural Analysis results.

Project field	Report value
Example / model	OB-EX-TRUSS-003 · Four-panel Howe truss
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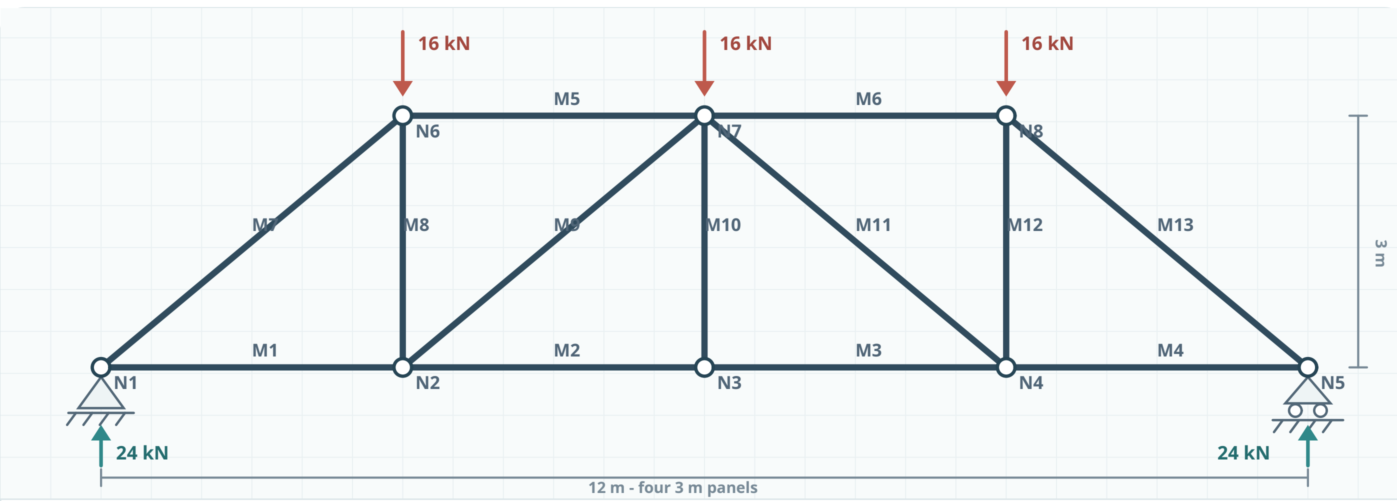
Last verified	2026-07-31

Analysis model and defined loads



Applied load | Support reaction | Pin at N1 - vertical roller at N5

LEFT VERTICAL REACTION	RIGHT VERTICAL REACTION	MAXIMUM COMPRESSION	MAXIMUM TENSION	CENTRE VERTICAL DISPLACEMENT
24.000 kN	24.000 kN	33.941 kN	32.000 kN	0.632 mm downward

Report role: This worked example documents the independent hand checks and their comparison with the analyzed model. The complete browser-generated Optimal Beam calculation package is a separate download: open the full solver package.

01 MODEL DEFINITION

Geometry, restraints, material and loading

A simply supported, four-panel Howe truss spans 12 m and is 3 m high. Three 16 kN downward loads act at the upper panel points. All members use the same steel modulus and cross-sectional area.

Span	Height	Panels	Supports	Material	Area
12 m	3 m	four 3 m panels	Pin + roller	E = 200 GPa	5,000 mm2

Node coordinates and supports

Node	X (m)	Y (m)	Support	Restrained translations
N1	0.0	0.0	Pin	UX, UY
N2	3.0	0.0	Free	-
N3	6.0	0.0	Free	-
N4	9.0	0.0	Free	-
N5	12.0	0.0	Vertical roller	UY
N6	3.0	3.0	Free	-
N7	6.0	3.0	Free	-
N8	9.0	3.0	Free	-

Support check: N1 is a pin that restrains UX and UY. N5 is a vertical roller that restrains UY only; horizontal translation at N5 remains free.

Applied nodal loads

Load	Node	FX (kN)	FY (kN)	MZ (kN m)
L1	N6	0.000	-16.000	0.000
L2	N7	0.000	-16.000	0.000
L3	N8	0.000	-16.000	0.000

02 STRUCTURAL IDEALIZATION

Member connectivity and analysis assumptions

The analytical model isolates the in-plane axial response of an ideal truss. The connectivity and assumptions below define the calculation.

Member	Connectivity	L (m)	Member	Connectivity	L (m)
M1	N1 to N2	3.000	M8	N6 to N2	3.000
M2	N2 to N3	3.000	M9	N2 to N7	4.243
M3	N3 to N4	3.000	M10	N7 to N3	3.000
M4	N4 to N5	3.000	M11	N7 to N4	4.243
M5	N6 to N7	3.000	M12	N8 to N4	3.000
M6	N7 to N8	3.000	M13	N8 to N5	4.243
M7	N1 to N6	4.243			

Included in the model

- Members are straight, prismatic and connected by ideal pins at the nodes.
- Loads are applied only at panel points, so the members carry idealized axial force.
- The material is linear elastic and all members use E = 200 GPa.
- All members use a uniform cross-sectional area of 5000 mm^2.
- The left support restrains global X and Y; the right support restrains global Y only.
- Member self-weight is omitted.

Outside the analysis scope

- Connection stiffness, gusset-plate behavior and joint eccentricity.
- Member buckling, yielding, fracture or code-based resistance checks.
- Geometric nonlinearity, P-Delta effects and large-displacement behavior.
- Out-of-plane response, lateral bracing and three-dimensional load paths.
- Construction tolerances, residual stress and fabrication effects.

Engineering boundary: Use the calculated forces to inform separate design checks. Verify buckling, yielding, connections, lateral bracing and code compliance for the real structure.

03 HAND CALCULATION

Global equilibrium and method of joints

Positive axial force denotes tension and negative axial force denotes compression. Global +Y is upward.

1. Determinacy

13 + 3 = 2(8) = 16

The count m + r = 2j satisfies the necessary condition for static determinacy. Inspection of the triangulated geometry and pin-and-roller support arrangement confirms that the truss is stable and has no internal or external mechanism.

2. Support reactions

Let A = N1 and B = N5. The geometry and loading are symmetric, so the two vertical reactions are equal.
ΣFy = 0: RAy + RBy - 3(16) = 0.
RAy = RBy = 48 / 2 = 24.000 kN.
ΣFx = 0: RAx = 0.000 kN.

The total calculated vertical reaction equals the total applied vertical load.

3. Zero-force members

M10 = 0.000 kN

At N3, the two bottom-chord members M2 and M3 are collinear and no external load acts at the joint. The non-collinear centre vertical M10 is therefore a zero-force member.

4. Joint N1

The end post M7 is inclined 45°.
ΣFy = 0: 24 + M7 sin 45° = 0, so M7 = -33.941 kN (compression).
ΣFx = 0: M1 + M7 cos 45° = 0, so M1 = +24.000 kN (tension).

Negative axial-force results denote compression; positive results denote tension.

03 HAND CALCULATION - CONTINUED

Method of sections and unit-load displacement

5. Section through M2, M5 and M9

Cut the second panel and take the left portion containing N1, N2 and N6. Assume the cut-member forces act in tension.
ΣM about N2 = 0: -24(3) - 3M5 = 0, so M5 = -24.000 kN.
ΣFy = 0: 24 - 16 + M9 sin 45° = 0, so M9 = -11.314 kN.
ΣFx = 0: M2 + M5 + M9 cos 45° = 0, so M2 = +32.000 kN.

The cut provides an independent check on representative chord and web-member forces.

6. Vertical displacement at N7 by the unit-load method

δN7 = Σ(Ni ni Li / EA) = 0.631529 mm downward

Apply a 1 kN downward virtual load at N7. Using the real member forces Ni, virtual member forces ni, member lengths Li and EA = 1,000,000 kN, the virtual-work sum gives the displacement in the direction of the unit load. In global coordinates, uy = -0.631529 mm.

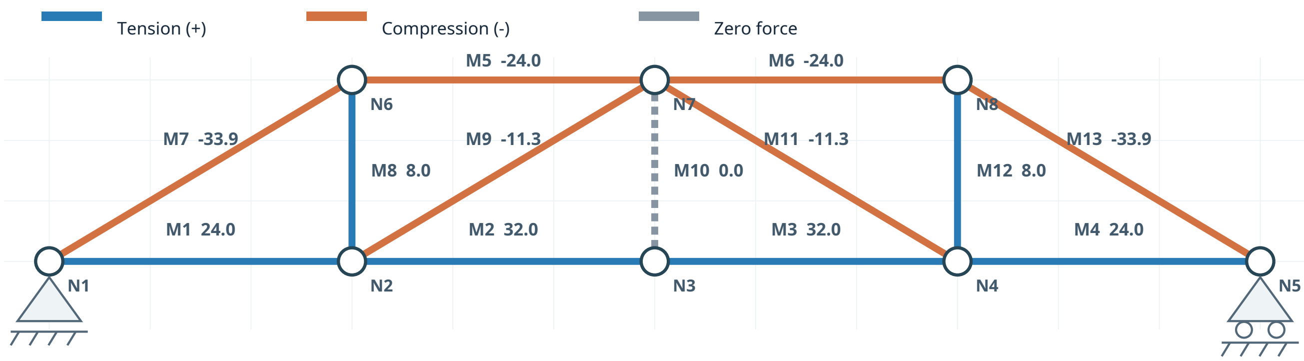
Unit-load contributions at N7

Members	Real N (kN)	Virtual n (kN)	L (m)	Subtotal (mm)
M1, M4	+24.000	+0.500000	3.000	0.072000
M2, M3	+32.000	+1.000000	3.000	0.192000
M5, M6	-24.000	-0.500000	3.000	0.072000
M7, M13	-33.941	-0.707107	4.243	0.203647
M8, M12	+8.000	+0.500000	3.000	0.024000
M9, M11	-11.314	-0.707107	4.243	0.067882
M10	0.000	0.000000	3.000	0.000000
Total				**0.631529**

04 OPTIMAL BEAM 2D SOLVER RESULTS

Axial-force results from Optimal Beam 2D

The axial forces below come directly from the analyzed result snapshot produced by the Optimal Beam 2D Structural Analysis Solver for this exact benchmark model. This sheet is a focused comparison summary; the separate 28-sheet calculation package is the complete browser-generated solver export.



Optimal Beam 2D Solver member results

Member	Connectivity	Axial force (kN)	State
M1	N1 to N2	24.000	Tension
M2	N2 to N3	32.000	Tension
M3	N3 to N4	32.000	Tension
M4	N4 to N5	24.000	Tension
M5	N6 to N7	-24.000	Compression
M6	N7 to N8	-24.000	Compression
M7	N1 to N6	-33.941	Compression
M8	N6 to N2	8.000	Tension
M9	N2 to N7	-11.314	Compression
M10	N7 to N3	0.000	Zero
M11	N7 to N4	-11.314	Compression
M12	N8 to N4	8.000	Tension
M13	N8 to N5	-33.941	Compression

Full Optimal Beam 2D Solver output: Download the separate 28-sheet calculation package generated from the 2D Structural Analysis workspace: four-panel-howe-truss-calculation-report.pdf.

05 INDEPENDENT VERIFICATION

Hand checks compared with Optimal Beam

Declared comparison checks

Check	Hand	Optimal Beam	Difference	Status
Left vertical reaction R_Ay	24.000000 kN	24.000000 kN	0.000%	**PASS**
Right vertical reaction R_By	24.000000 kN	24.000000 kN	0.000%	**PASS**
Bottom chord M2	32.000000 kN	32.000000 kN	0.000%	**PASS**
Top chord M5	-24.000000 kN	-24.000000 kN	0.000%	**PASS**
Diagonal M9	-11.313708 kN	-11.313708 kN	0.000%	**PASS**
Zero-force vertical M10	0.000000 kN	0.000000 kN	0.000%	**PASS**
Centre vertical displacement N7	-0.631529 mm	-0.631529 mm	0.000%	**PASS**
Vertical equilibrium	48.000 kN load	48.000 kN reaction	0.000%	**PASS**

Verification result: All declared checks passed. Relative equilibrium residual: 5.093e-11. Last model verification: 2026-07-31.

Calculation traceability

Package field	Report value
Generated by	Optimal Beam worked-example report generator
Model / scenario	Four-panel Howe truss · All applied loads
Contents	28 sheets: model, inputs, diagnostics, reactions, displacements, diagrams and member results
Download	https://optimalbeam.com/output/pdf/four-panel-howe-truss-calculation-report.pdf

Internal file paths and code fingerprints are retained in automated release tests, not shown as user-facing engineering report content.

06 MODELING REVIEW

Common mistakes and responsibility boundary

The following changes alter the analytical problem rather than merely changing presentation.

01 Applying a distributed load directly to an ideal truss member instead of converting it to panel-point loads.	02 Drawing the interior diagonals in the Pratt direction while intending to model a Howe truss.
03 Assuming the Howe diagonals are in tension under this gravity load instead of checking their force sign.	04 Missing the zero-force centre vertical M10 at the unloaded bottom joint.
05 Restraining both supports horizontally and unintentionally introducing an extra reaction.	06 Using the axial-force result as a strength, buckling or connection design check.

Analysis statement

This report documents an ideal pin-jointed truss benchmark. It calculates reactions, member axial forces and elastic displacement. It does not design members, connections or bracing and does not evaluate yielding, fracture, buckling or code compliance.

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Analysis scope: First-order linear elastic 2D analysis
Editable model: optimalbeam.com/2d-structural-analysis/?example=howe-truss
Full solver package: https://optimalbeam.com/output/pdf/four-panel-howe-truss-calculation-report.pdf