

2D FRAME AND TRUSS ANALYSIS

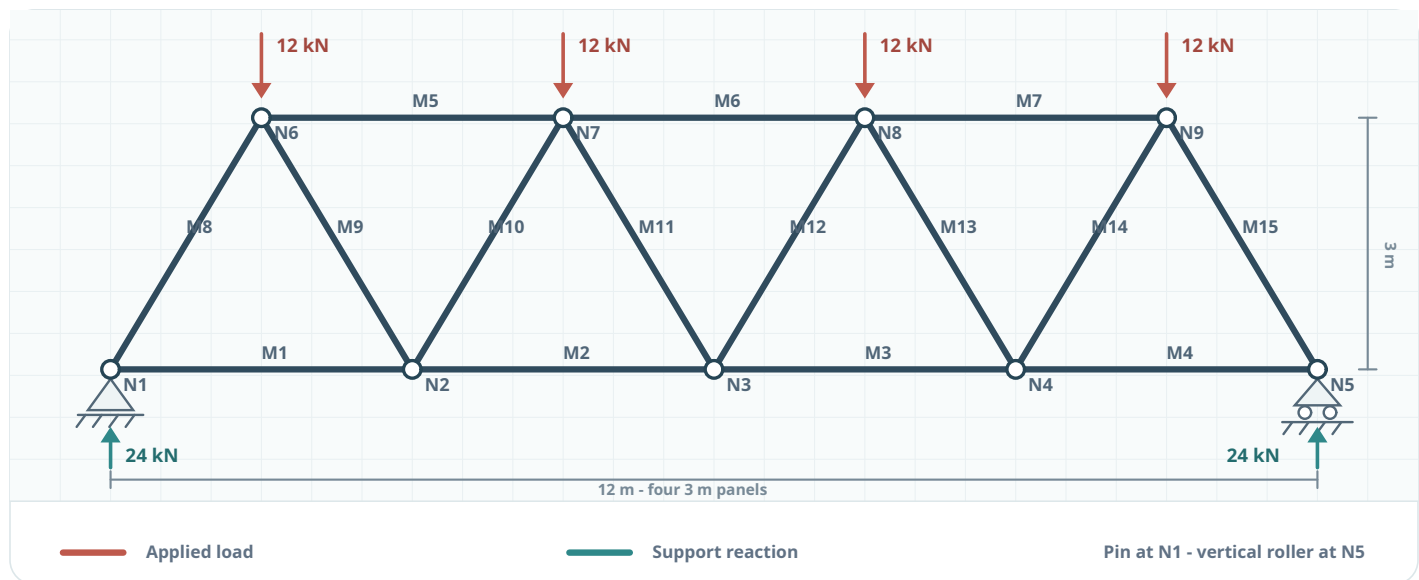
Warren Truss Analysis: Worked Example

Four-panel Warren truss

Hand calculation, independent checks and comparison with the Optimal Beam 2D Structural Analysis results.

Project field	Report value
Example / model	OB-EX-TRUSS-002 · Four-panel Warren truss
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Last verified	2026-07-27

Analysis model and defined loads



LEFT VERTICAL REACTION 24.000 kN	RIGHT VERTICAL REACTION 24.000 kN	MAXIMUM COMPRESSION 26.833 kN	MAXIMUM TENSION 24.000 kN	CENTRE VERTICAL DISPLACEMENT 0.453 mm downward
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Report role: This worked example documents the independent hand checks and their comparison with the analyzed model. The complete browser-generated Optimal Beam calculation package is a separate download: [open the full solver package](#).

01 MODEL DEFINITION

Geometry, restraints, material and loading

A simply supported, four-panel Warren truss spans 12 m and is 3 m high. Four 12 kN downward loads act at the upper panel points. All members use the same steel modulus and cross-sectional area.

Span	Height	Panels	Supports	Material	Area
12 m	3 m	4 bottom-chord panels at 3 m	Pin + roller	E = 200 GPa	5,000 mm ²

Node coordinates and supports

Node	X (m)	Y (m)	Support	Restrained translations
N1	0.0	0.0	Pin	UX, UY
N2	3.0	0.0	Free	-
N3	6.0	0.0	Free	-
N4	9.0	0.0	Free	-
N5	12.0	0.0	Vertical roller	UY
N6	1.5	3.0	Free	-
N7	4.5	3.0	Free	-
N8	7.5	3.0	Free	-
N9	10.5	3.0	Free	-

Support check: N1 is a pin that restrains UX and UY. N5 is a vertical roller that restrains UY only; horizontal translation at N5 remains free.

Applied nodal loads

Load	Node	FX (kN)	FY (kN)	MZ (kN m)
L1	N6	0.000	-12.000	0.000
L2	N7	0.000	-12.000	0.000
L3	N8	0.000	-12.000	0.000
L4	N9	0.000	-12.000	0.000

02 STRUCTURAL IDEALIZATION

Member connectivity and analysis assumptions

The analytical model isolates the in-plane axial response of an ideal truss. The connectivity and assumptions below define the calculation.

Member	Connectivity	L (m)	Member	Connectivity	L (m)
M1	N1 to N2	3.000	M9	N6 to N2	3.354
M2	N2 to N3	3.000	M10	N2 to N7	3.354
M3	N3 to N4	3.000	M11	N7 to N3	3.354
M4	N4 to N5	3.000	M12	N3 to N8	3.354
M5	N6 to N7	3.000	M13	N8 to N4	3.354
M6	N7 to N8	3.000	M14	N4 to N9	3.354
M7	N8 to N9	3.000	M15	N9 to N5	3.354
M8	N1 to N6	3.354			

Included in the model

- Members are straight, prismatic and connected by ideal pins at the nodes.
- Loads are applied only at panel points, so the members carry idealized axial force.
- The material is linear elastic and all members use $E = 200 \text{ GPa}$.
- All members use a uniform cross-sectional area of 5000 mm^2 .
- The left support restrains global X and Y; the right support restrains global Y only.
- Member self-weight is omitted.

Outside the analysis scope

- Connection stiffness, gusset-plate behavior and joint eccentricity.
- Member buckling, yielding, fracture or code-based resistance checks.
- Geometric nonlinearity, P-Delta effects and large-displacement behavior.
- Out-of-plane response, lateral bracing and three-dimensional load paths.
- Construction tolerances, residual stress and fabrication effects.

Engineering boundary: Use the calculated forces to inform separate design checks. Verify buckling, yielding, connections, lateral bracing and code compliance for the real structure.

03 HAND CALCULATION

Global equilibrium and method of joints

Positive axial force denotes tension and negative axial force denotes compression. Global +Y is upward.

1. Determinacy

$$15 + 3 = 2(9) = 18$$

The count $m + r = 2j$ satisfies the necessary condition for static determinacy. Inspection of the repeating triangular geometry and pin-and-roller support arrangement confirms that the truss is stable and has no internal or external mechanism.

2. Support reactions

Let $A = N1$ and $B = N5$. The geometry and loading are symmetric, so the two vertical reactions are equal.
 $\Sigma F_y = 0$: $RA_y + RB_y - 4(12) = 0$.
 $RA_y = RB_y = 48 / 2 = 24.000 \text{ kN}$.
 $\Sigma F_x = 0$: $RA_x = 0.000 \text{ kN}$.

The total calculated vertical reaction equals the total applied vertical load.

3. Zero-force members

$$M11 = M12 = 0.000 \text{ kN}$$

M11 and M12 are mirror-symmetric about midspan, so their axial forces are equal. At unloaded joint N3, their horizontal components cancel while their vertical components add. Because M2 and M3 are horizontal and no external vertical load acts at N3, $\Sigma F_y = 2F \sin \theta = 0$ and both centre diagonals carry zero force.

4. Joint N1

The outer diagonal M8 has a 1.5 m run and 3 m rise, so $\theta = 63.435^\circ$, $\sin \theta = 0.894427$ and $\cos \theta = 0.447214$.
 $\Sigma F_y = 0$: $24 + M8 \sin \theta = 0$, so $M8 = -26.833 \text{ kN}$ (compression).
 $\Sigma F_x = 0$: $M1 + M8 \cos \theta = 0$, so $M1 = +12.000 \text{ kN}$ (tension).

Negative axial-force results denote compression; positive results denote tension.

03 HAND CALCULATION - CONTINUED

Method of sections and unit-load displacement

5. Section through M2, M5 and M10

Cut the second panel and take the left portion containing N1, N2 and N6. Assume the cut-member forces act in tension.
 $\Sigma M \text{ about } N2 = 0: -24(3) + 12(1.5) - 3M5 = 0$, so $M5 = -18.000 \text{ kN}$.
 $\Sigma Fy = 0: 24 - 12 + M10 \sin \theta = 0$, so $M10 = -13.416 \text{ kN}$.
 $\Sigma Fx = 0: M2 + M5 + M10 \cos \theta = 0$, so $M2 = +24.000 \text{ kN}$.

The cut provides an independent check on representative chord and web-member forces.

6. Vertical displacement at N3 by the unit-load method

$\delta N3 = \Sigma(Ni \ ni \ Li / EA) = 0.453246 \text{ mm downward}$

Apply a 1 kN downward virtual load at N3. Using the real member forces Ni , virtual member forces ni , member lengths Li and $EA = 1,000,000 \text{ kN}$, the virtual-work sum gives the displacement in the direction of the unit load. In global coordinates, $uy = -0.453246 \text{ mm}$.

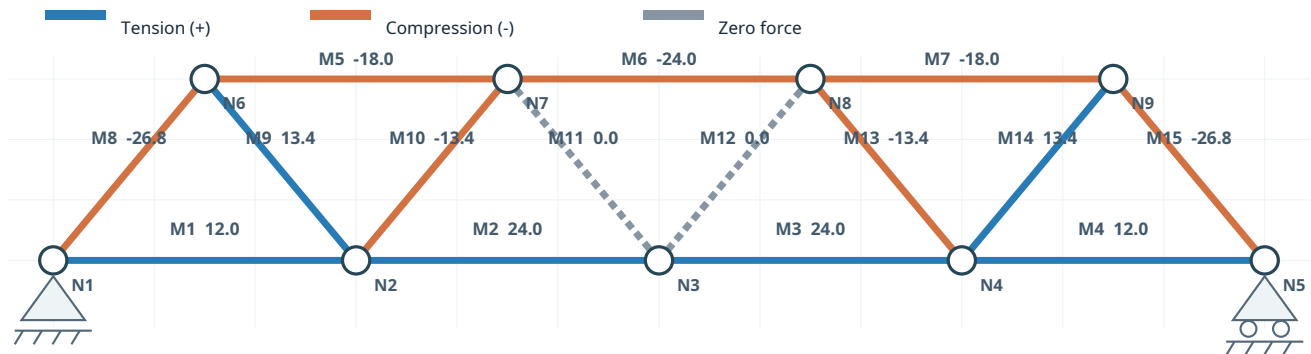
Unit-load contributions at N3

Members	Real N (kN)	Virtual n (kN)	L (m)	Subtotal (mm)
M1, M4	+12.000	+0.250000	3.000	0.018000
M2, M3	+24.000	+0.750000	3.000	0.108000
M5, M7	-18.000	-0.500000	3.000	0.054000
M6	-24.000	-1.000000	3.000	0.072000
M8, M15	-26.833	-0.559017	3.354	0.100623
M9, M14	+13.416	+0.559017	3.354	0.050312
M10, M13	-13.416	-0.559017	3.354	0.050312
M11, M12	0.000	+0.559017	3.354	0.000000
Total				0.453246

04 OPTIMAL BEAM 2D SOLVER RESULTS

Axial-force results from Optimal Beam 2D

The axial forces below come directly from the analyzed result snapshot produced by the Optimal Beam 2D Structural Analysis Solver for this exact benchmark model. This sheet is a focused comparison summary; the separate 32-sheet calculation package is the complete browser-generated solver export.



Optimal Beam 2D Solver member results

Member	Connectivity	Axial force (kN)	State
M1	N1 to N2	12.000	Tension
M2	N2 to N3	24.000	Tension
M3	N3 to N4	24.000	Tension
M4	N4 to N5	12.000	Tension
M5	N6 to N7	-18.000	Compression
M6	N7 to N8	-24.000	Compression
M7	N8 to N9	-18.000	Compression
M8	N1 to N6	-26.833	Compression
M9	N6 to N2	13.416	Tension
M10	N2 to N7	-13.416	Compression
M11	N7 to N3	0.000	Zero
M12	N3 to N8	0.000	Zero
M13	N8 to N4	-13.416	Compression
M14	N4 to N9	13.416	Tension
M15	N9 to N5	-26.833	Compression

Full Optimal Beam 2D Solver output: Download the separate 32-sheet calculation package generated from the 2D Structural Analysis workspace: [four-panel-warren-truss-calculation-report.pdf](#).

05 INDEPENDENT VERIFICATION

Hand checks compared with Optimal Beam

Declared comparison checks

Check	Hand	Optimal Beam	Difference	Status
Left vertical reaction R_Ay	24.000000 kN	24.000000 kN	0.000%	PASS
Right vertical reaction R_By	24.000000 kN	24.000000 kN	0.000%	PASS
Bottom chord M2	24.000000 kN	24.000000 kN	0.000%	PASS
Top chord M5	-18.000000 kN	-18.000000 kN	0.000%	PASS
Diagonal M10	-13.416408 kN	-13.416408 kN	0.000%	PASS
Zero-force diagonal M11	0.000000 kN	0.000000 kN	0.000%	PASS
Centre vertical displacement N3	-0.453246 mm	-0.453246 mm	0.000%	PASS
Vertical equilibrium	48.000 kN load	48.000 kN reaction	0.000%	PASS

Verification result: All declared checks passed. Relative equilibrium residual: 7.276e-11. Last model verification: 2026-07-27.

Calculation traceability

Package field	Report value
Generated by	Optimal Beam worked-example report generator
Model / scenario	Four-panel Warren truss · All applied loads
Contents	32 sheets: model, inputs, diagnostics, reactions, displacements, diagrams and member results
Download	https://optimalbeam.com/output/pdf/four-panel-warren-truss-calculation-report.pdf

Internal file paths and code fingerprints are retained in automated release tests, not shown as user-facing engineering report content.

06 MODELING REVIEW

Common mistakes and responsibility boundary

The following changes alter the analytical problem rather than merely changing presentation.

01 Applying a distributed load directly to an ideal truss member instead of converting it to panel-point loads.	02 Assuming every Warren diagonal alternates force sign without checking the actual geometry and load positions.
03 Missing the two zero-force centre diagonals created by this symmetric load case.	04 Drawing crossing members without creating a shared node where a connection is intended.
05 Restraining both supports horizontally and unintentionally introducing an extra reaction.	06 Using the axial-force result as a strength, buckling or connection design check.

Analysis statement

This report documents an ideal pin-jointed truss benchmark. It calculates reactions, member axial forces and elastic displacement. It does not design members, connections or bracing and does not evaluate yielding, fracture, buckling or code compliance.

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Analysis scope: First-order linear elastic 2D analysis

Editable model: optimalbeam.com/2d-structural-analysis/?example=warren-truss

Full solver package: <https://optimalbeam.com/output/pdf/four-panel-warren-truss-calculation-report.pdf>